NEW STUDIES OF EMITTANCE GROWTH AND CORRECTION TECHNIQUES FOR THE TESLA LINAC

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Abstract

Spectral analysis has been used to study emittance growth due to chromatic effects in future linear colliders. Correction techniques can be dynamically evaluated by this method. This spectral formalism has been applied to the TESLA linac.

1 INTRODUCTION

Displacements of focusing magnets will dilute the beam emittance in future linear colliders through dispersive effects. The final dispersive error can be found with the help of the spectral analysis. This formalism allows to study the effects of static initial misalignments, as well as the effects of displacements produced by ground motion, which is adequately described by the 2-D power spectrum $P(\omega, k)$ [1]. The chromatic dilution is then given by an integral involving the power spectrum of the quadrupole displacements and a spectral response function describing the transport line. The effectiveness of correction techniques, envisaged in future linear colliders to recover the small required emittance, can be also evaluated by the spectral approach [2], provided that the correlations between space harmonics are correctly taken into account. The results of the "one-to-one" correction and the "adaptive alignment" [3] method are given for the TESLA linac for illustration.

2 SPECTRAL ANALYSIS OF CHROMATIC DILUTION

Let $x_i(t) = x(t, s_i)$ be the transverse position of quadrupoles of a linac, relatively to a reference line, s_i the longitudinal position. The incoming beam angle and position are zero, the reference line passes through some element, placed at the entrance. The dispersion, linear term, is

$$\eta_x(t) = \sum_{i=1}^{N} d_i\, x_i(t)$$

Here d_i is the first derivative of the beam dispersion at the exit of the linac with respect to the displacement of the quadrupole i, N is the total number of quadrupoles. In thin lens approximation, in linear order

$$d_i = K_i\,(r^i_{12} - t^i_{126})$$

where K_i is r_{21} of the quadrupole matrix, r^i_{12} and t^i_{126} are the elements of the first and the second order transfer matrices from the i-th quadrupole to the exit.

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While $\langle \eta_x(t)\rangle$, averaged on realizations, is zero, the mean squared value gives the dispersive error:

$$\langle \eta_x^2(t)\rangle = \sum\sum d_i d_j\,\langle x_i(t) x_j(t)\rangle$$

One can introduce the spatial harmonics $x(t, k)$

$$x(t,k) = \int_{-\mathcal{L}/2}^{\mathcal{L}/2} x(t,s) e^{-\mathrm{i}ks}\, ds$$

and by use of the back transformation

$$x(t,s) = \int_{-\infty}^{\infty} x(t,k)\left(e^{\mathrm{i}ks} - 1\right)\frac{dk}{2\pi}$$

which ensures that at the entrance $x(t, s = 0) = 0$, one can find $\langle \eta_x^2(t)\rangle$. For initial misalignment or (and) ground motion all spatial harmonics are independent. We have then

$$\langle \eta_x^2(t)\rangle = \int_{-\infty}^{\infty} P(t,k)\, G(k)\,\frac{dk}{2\pi}$$

Here $G(k)$ is the so called spectral response function

$$G(k) = g_c^2(k) + g_s^2(k) \tag{1}$$

with

$$g_c(k) = \sum_{i=1}^{N} d_i(\cos(ks_i) - 1)\,,\; g_s(k) = \sum_{i=1}^{N} d_i \sin(ks_i)$$

The spatial power spectrum of displacements $x(t, s)$ is

$$P(t,k) = \lim_{\mathcal{L}\to\infty} 1/\mathcal{L}\; x(t,k)x^*(t,k)$$

It can be easily found as far as initial misalignment or ground motion are concerned. Assuming that focusing elements are aligned at $t = 0$ and then are moved by ground motion, the evolution of the power spectrum is [1]:

$$P(t,k) = \int_{-\infty}^{\infty} P(\omega,k)\;\; 2\left[1 - \cos(\omega t)\right]\frac{d\omega}{2\pi}$$

Here the 2-D power spectrum $P(\omega, k)$ characterizes ground motion properties, including both spatial and temporal correlation information. Several models of $P(\omega, k)$ have been discussed in [1]. The diffusive ground motion, leading to large displacements after long time intervals, is described by the "ATL law" [4]. Its power spectrum $P(\omega, k)$ is:

$$P(\omega,k) = A/(\omega^2 k^2)$$

Typically $A = 10^{-5}\mu\mathrm{m}^2\mathrm{s}^{-1}\mathrm{m}^{-1}$. Though any $P(\omega, k)$ can be considered [2], we use only "ATL" motion in the paper.

Correction procedures may introduce correlation of phases between harmonics with different k. In a regular linac with constant spacing L, the correction techniques, considered in this paper, introduce phase correlations only between harmonics k and $\tilde{k} = k_{\max} - k$. The mean squared dispersion is then [2]:

$$\langle \eta_x^2(t)\rangle = 2\int_{k_{\min}}^{k_{\max}} \left(P(t,k)G(k) + \mathcal{P}(t,k)\mathcal{G}(k)\right)\frac{dk}{2\pi} \tag{2}$$

It contains the self correlation spectrum

$$\mathcal{P}(t,k) = \Re \lim_{\mathcal{L}\to\infty} 1/\mathcal{L}\; x(t,k)x(t,\tilde{k})$$

and the new spectral function

$$\mathcal{G}(k) = g_c(k)g_c(\tilde{k}) - g_s(k)g_s(\tilde{k})$$

The integral (2) is taken on the allowed band for the regular linac $k_{\min} < |k| < k_{\max}$, where $k_{\max} = \pi/L$, $k_{\min} = 2\pi/(N\,L)$ (for the finite linac the upper limit is $k_{\max} - k_{\min}$). The ground motion, which can have any k, has to be correctly redistributed within the allowed band.

In short, the spectral response functions $G(k)$ and $\mathcal{G}(k)$ describe the properties of the focusing channel, while the power $P(t,k)$ and the self correlation $\mathcal{P}(t,k)$ spectra depend on the applied method of correction, initial misalignment and ground motion.

3 "ONE-TO-ONE" TECHNIQUES

The "one-to-one" algorithm consists in zeroing the BPM measurements. This can be done by steering the beam by means of dipole correctors or by moving the misaligned quadrupoles towards the beam.

3.1 "One-to-one" by steering

If the i-th quadrupole is misaligned, three angles are needed to re-align the beam. The equivalent quadrupole displacements, to be subtracted from their initial positions, are

$$\Delta x_i = -2x_i/(LK_i)\,,\ \Delta x_{i+1} = \Delta x_{i-1} = -x_i/(LK_i)$$

For a regular FODO lattice, with $K_i = -K_{i+1}$, a k-th harmonics of the initial misalignment produces two harmonics of quadrupole displacements after the correction: k-th and $(k_{\max} - k)$-th with opposite phases. Finally, the power spectrum of quadrupole displacements after correction, with "ATL" ground motion, is [2]:

$$\begin{aligned} P(t,k) &= L(\sigma_{\rm ini}^2 + \sigma_{\rm err}^2)\left(1 + \tilde{r}_2^2\right) \qquad (3)\\ &+ At\left(1/k^2 + 1/k_{\max}^2 + \tilde{r}_2^2(1/\tilde{k}^2 + 1/k_{\max}^2)\right) \end{aligned}$$

where $r_2 = 2(1-\cos(kL))/(LK)$, $\tilde{r}_2 = r_2(\tilde{k}) = 2(1+\cos(kL))/(LK)$. The self correlation is

$$\begin{aligned} \mathcal{P}(t,k) &= 4L(\sigma_{\rm ini}^2 + \sigma_{\rm err}^2)/(LK) \qquad (4)\\ &+ At\left(r_2(1/k^2 + 1/k_{\max}^2) + \tilde{r}_2(1/\tilde{k}^2 + 1/k_{\max}^2)\right) \end{aligned}$$

Here $\sigma_{\rm err}$ is the total rms BPM error, including both BPM offset and resolution ($\sigma_{\rm err}^2 = \sigma_{\rm off}^2 + \sigma_{\rm res}^2$). We assume Gaussian initial misalignments and BPM errors. For illustration, Fig.1 shows spectra in comparson with simulations. All examples refer to a model of the TESLA linac, when $N = 618$, $L = 24.4$ m, phase advance $\mu = 60^o$, initial energy $\gamma_{\rm ini} = 6000$, $\gamma_{\rm fin} = 5\,10^5$, beta function at the exit $\beta_N = 28.17$ m.

The dispersion can be found by use of (3,4), provided that injection conditions are correctly specified [2]. Alternatively, one can show that for the "one-to-one" corrections the dispersive error can be written

$$\langle \eta_x^2(t)\rangle = 2\int_{k_{\min}}^{k_{\max}} \hat{P}(t,k)\,\hat{G}(k)\,\frac{dk}{2\pi} \qquad (5)$$

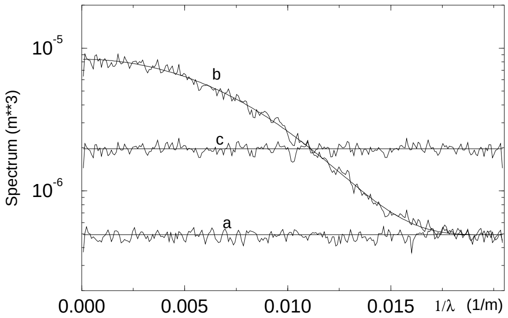

Figure 1: Initial (a), final (b) and self correlation (c) spectra for one to one correction by steering, $\sigma_{\rm ini} = 100\mu$m. All spectra on pictures doubled in comparing with formulae.

where $\hat{G}(k)$ and $\hat{P}(t,k)$ are the effective spectral response function and the effective spectrum of quadrupole displacements before correction respectively. The $\hat{G}(k)$ is built with new dispersive coefficients [2]

$$\hat{d}_i = d_i + (2d_i + d_{i+1} + d_{i-1})/(LK_i)$$

and $\hat{P}(t,k)$ is given by

$$\hat{P}(t,k) = L(\sigma_{\rm ini}^2 + \sigma_{\rm err}^2) + At\left(1/k^2 + 1/k_{\max}^2\right)$$

It is useful to note that if $\gamma_{\rm ini} = \gamma_{\rm fin}$, then $\hat{d}_i = -K_i r_{12}^i$.

An example of analytical results (5) together with simulations by particle tracking is shown on Fig.2. The analytical results exhibits the following dependencies before and after correction, respectively:

$$\langle \eta_x^2\rangle \approx (\sigma_{\rm ini}^2 + 0.5AtL)\,0.038N^3$$

$$\langle \eta_x^2\rangle \approx (\sigma_{\rm ini}^2 + \sigma_{\rm err}^2 + 1.1\,AtL)\,1.1N$$

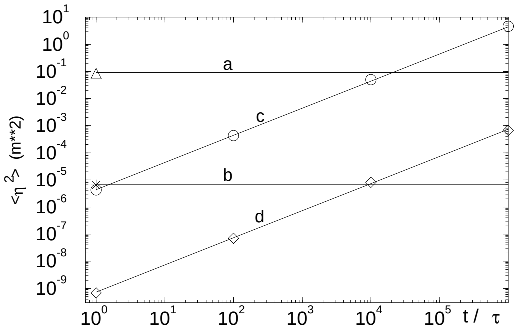

Figure 2: Dispersive error for the "one-to-one" correction by steering, a) and b) $\sigma_{\rm ini} = 100\ \mu$m; c) and d) $A\tau L = 10^{-12}$ m^2, before and after correction.

3.2 "One-to-one" by quadrupole moving

The beam will now be passed through the center of the i-th BPM by moving the i-th quadrupole. The resulted quadrupole misalignments depend only on the total BPM errors. We can show [2] that the power spectrum of quadrupole displacement after correction is

$$P(k) = L\sigma_{\rm err}^2\left(1 + \frac{(KL)^2}{4(1-\cos(kL))^2}\right) \qquad (6)$$

The self correlation spectrum is then

$$\mathcal{P}(k) = L\sigma_{\rm err}^2\frac{KL}{\sin^2(kL)} \qquad (7)$$

Unlike the steering method, the power spectrum grows for small k as $1/k^4$, showing a smooth deviation of the quadrupoles line from its original position.

These spectra can be used to find dispersion, provided that injection conditions are correctly specified [2]. In the same way as before, one can alternatively introduce new coefficients [2]

$$\hat{d}_i = -d_i + K_i \sum_{j=i+1}^{N} d_j (s_j - s_i)$$

to build the effective $\hat{G}(k)$. The effective spectrum is $\hat{P}(k) = L\sigma_{\rm err}^2$ (initial misalignment and ground motion are vanished by correction, the effect of ground motion during correction assumed to be small). The dispersive error is then given by (5).

The analytical results, confirmed by tracking, are:

$$\langle \eta_x^2 \rangle \approx \sigma_{\rm err}^2 \, 3.8N$$

The so-called "shunt" method can be described by the same equations. It consists in moving a quadrupole in such a way that changing of its strength does not produce beam shift in the next BPM. If the relative strength change is $\delta_K = \delta K/K$ then the precision of cancelation of the BPM offset is $\sigma_{\rm res}/(KL\,\delta_K)$. The spectra of the quadrupoles after alignment are given by Eqs.(6, 7) where now $\sigma_{\rm err}^2 = \sigma_{\rm res}^2(1 + 1/(KL\,\delta_K)^2)$.

4 THE "ADAPTIVE ALIGNMENT"

The "adaptive alignment" algorithm [3] calculates from the readings a_i of three neighboring BPMs the change of position of the central quadrupole

$$\Delta x_i = c_0 \left(a_{i+1} + a_{i-1} - a_i(2 + K_i L)\right)/3$$

The coefficient c_0 controls the velocity of convergence of the algorithm. This procedure is repeated iteratively. If only i-th quadrupole is misaligned and BPMs are perfect, then the corrections at the first iteration are:

$$\Delta x_{i-1} = \Delta x_{i+1} = -c_0 x_i/3\,, \quad \Delta x_i = 2c_0 x_i/3$$

The power spectrum after n-th iteration at $t = n\Delta t$ is

$$P_{(n)}(k) = r_1^{2n} L\sigma_{\rm ini}^2 + A\Delta t \left(\frac{1}{k^2} + \frac{1}{k_{\rm max}^2}\right) r_1^2 \frac{1 - r_1^{2n}}{1 - r_1^2}$$
$$+ L(r_3^2 + \tilde{r}_4^2)\left(\sigma_{\rm off}^2 \frac{(1 - r_1^n)^2}{(1 - r_1)^2} + \sigma_{\rm res}^2 \frac{(1 - r_1^{2n})}{(1 - r_1^2)}\right)$$

The self correlation is

$$\mathcal{P}_{(n)}(k) = L(r_3\tilde{r}_4 + r_4\tilde{r}_3)\left(\sigma_{\rm off}^2 \frac{(1 - r_1^n)(1 - \tilde{r}_1^n)}{(1 - r_1)(1 - \tilde{r}_1)} + \sigma_{\rm res}^2 \frac{(1 - (r_1\tilde{r}_1)^n)}{(1 - r_1\tilde{r}_1)}\right)$$

Here $r_1(k) = 1 - 2c_0(1 - \cos(kL))/3$, $r_3(k) = -2c_0(1 - \cos(kL))/3$, $r_4(k) = -c_0 KL/3$.

Even at $n = 1$, some harmonics, for which $r_1 = 0$, is damped completely if BPMs are perfect (see Fig.3). If $c_0 < 3/2$ the algorithm converge. The optimum value $c_0 = 1$.

The analytical results (2) in comparing with simulations (particle tracking) are shown in Fig.4 (for $c_0 = 1$). The equilibrium value of dispersion error is approximately

$$\langle \eta_x^2 \rangle_\infty \approx (\sigma_{\rm res}^2 + 0.0054\sigma_{\rm off}^2 + 0.83A\Delta t L)\, 0.059N^3$$

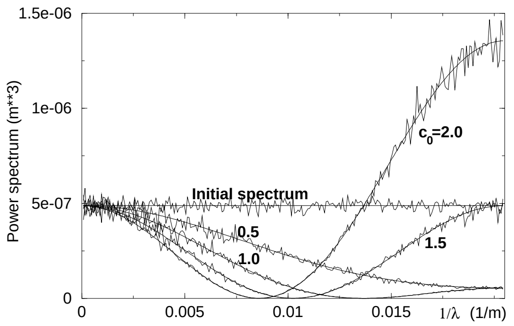

Figure 3: Power spectrum after first iteration of the "adaptive alignment" for different c_o, $\sigma_{\rm ini} = 100\mu$m.

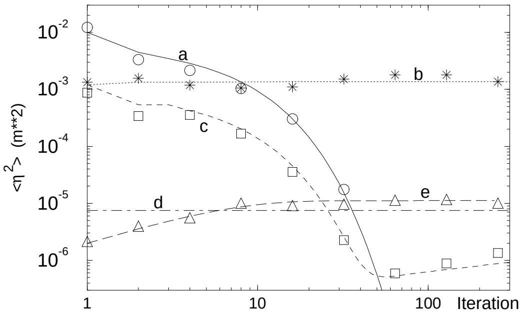

Figure 4: Dispersive error for "adaptive alignment". a) $\sigma_{\rm ini} = 100\mu$m; b) $\sigma_{\rm res} = 10\mu$m; c) $\sigma_{\rm off} = 10\mu$m; d) the limit at $n \to \infty$ of the case c); e) $A\Delta t L = 10^{-12}$ m^2.

5 CONCLUSION

The spectral analysis allowed the estimation of the chromatic dilution in future colliders with static initial misalignments and with the effects of ground motion. This formalism was applied to the TESLA linac, where two different types of correction were studied in detail for illustration. Numerical simulations and analytical results are in good agreement. A regular linac, having a constant spacing of the focusing elements, is the only limitation we saw in this spectral approach.

6 REFERENCES

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